Part 1

Chapter 1 Sets and Sequences

<u>Set</u>: A collection of "well defined" objects.

<u>eg</u>: $A = \{a_1, a_2, a_3\}$ $a_2 \in A$

set of integer $\mathbb{Z} = \{0, \pm 1, \pm 2, \cdots\}$

" " natural no $\mathbb{N} = \{1, 2, 3, 4, \cdots\}$

" " real no. $\mathbb{R} = \{x : -\infty < x < \infty\}$

Basic Terminologies

1) Null set : ϕ (contains "nothing")
   ["nothing" is an "well defined" obj]
2) finite / Infinite set (# elements $< \infty$ / $= \infty$)
3) "Implies" ($A \Rightarrow B$ means if A then B)
   $\Leftrightarrow$
   $\wedge$ (and) $\vee$ (or)

4) Universal set : [relative notion]

5) <u>Subset / Superset</u> $(\subseteq)$

If $x \in A \Rightarrow x \in B$ then $A \subseteq B$

<u>eg</u>: $\mathbb{N} \subseteq \mathbb{Z}$

6) <u>Union / Intersection</u>

$A \cup B = \{x \in A \vee x \in B\}$

$A \cap B = \{x \in A \wedge x \in B\}$

<u>Finite union / intersection</u> $\bigcup_{i=1}^{n} A_i \;\; || \;\; \bigcap_{i=1}^{n} A_i$

Arbitrary Union / Intersection

~~The~~ consider collection of subsets

$\{ A_\lambda : \lambda \in \Lambda \}$ of a set S

Λ = index set

$\bigcup_{\lambda \in \Lambda} A_\lambda$ = union

$\bigcap_{\lambda \in \Lambda} A_\lambda$ = intersection

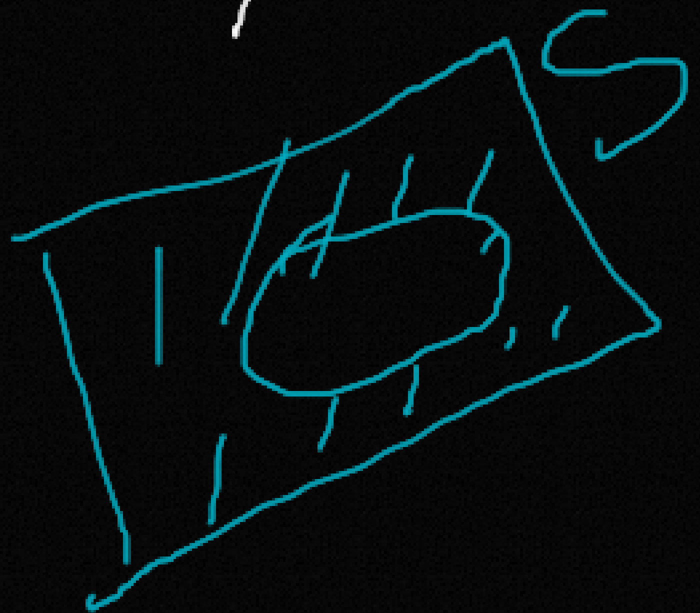
ex: Let $A_i = \{ i \}$
if $\Lambda = \mathbb{N} = \{1, 2, 3, \ldots\}$
then $\mathbb{N} = \bigcup_{\lambda \in \Lambda} A_\lambda = \bigcup_{i=1}^{\infty} A_i$

Disjoint set

A and B disjoint if $A \cap B = \phi$

$A_1, A_2, \dots, A_n$ disjoint if

$$\bigcap_{i=1}^{n} A_i = \phi$$



Complement

If A is a subset of S then $A^c = \{x : (x \in S) \wedge (x \notin A)\}$

eg: $A = (0, \infty)$, $A^c = (-\infty, 0)$

Difference: Let $A \subseteq S$, $B \subseteq S$,

then $A \setminus B = \{x : x \in A \wedge x \notin B\}$

$= A \cap B^c$

$A \setminus B = \{1, 4\}$

$\phi, \{\phi\}$

Symmetric Difference: $A \triangle B$

$= (A \setminus B) \cup (B \setminus A)$

eg: $A \triangle B = \{1, 4, 6\}$

Power of a set :

The collection of all subsets of A is called the power set of A and denoted as $P(A)$

If $\# A = n$ then $\# P(A) = 2^n$

eg : If $A = \{2, 3\}$. $P(A) = \{\phi, \{2\}, \{3\}, \{2, 3\}\}$

Set Algebra

Commutative : $A \cup B = B \cup A$

$A \cap B = B \cap A$

Associative : $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

Distributive : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan : $(A \cup B)^c = A^c \cap B^c$

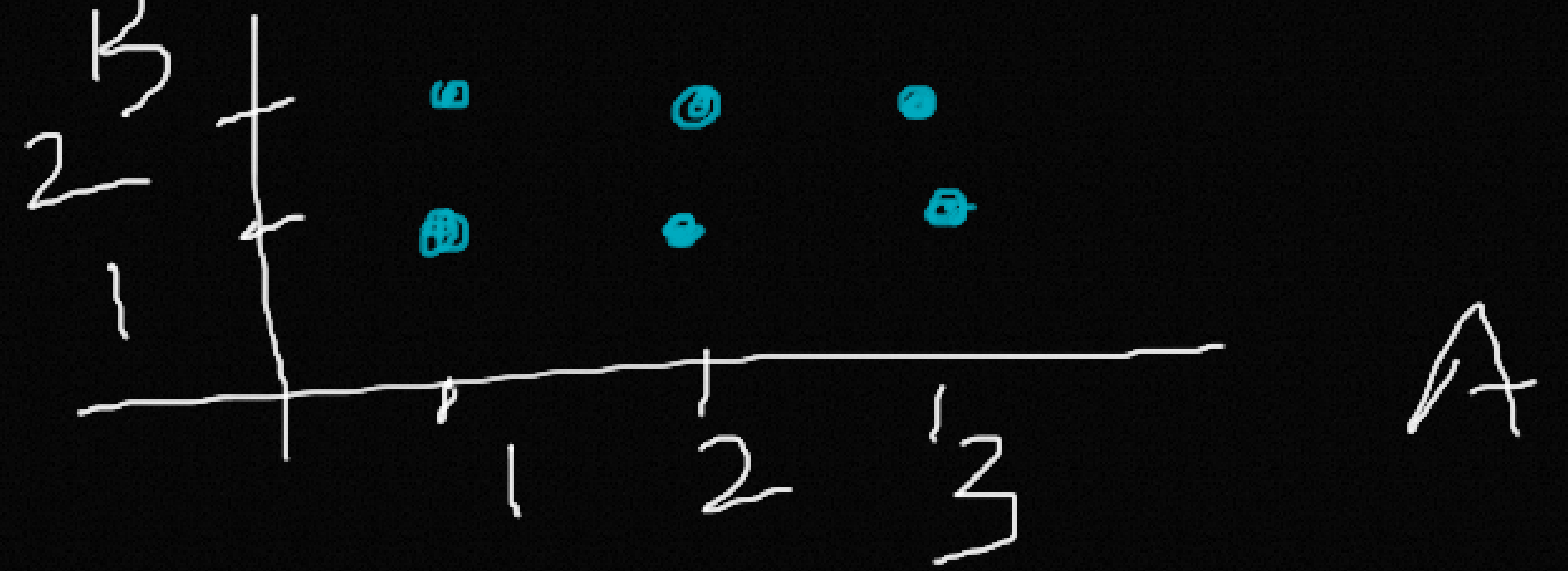
$(A \cap B)^c = A^c \cup B^c$

Cartesian Product

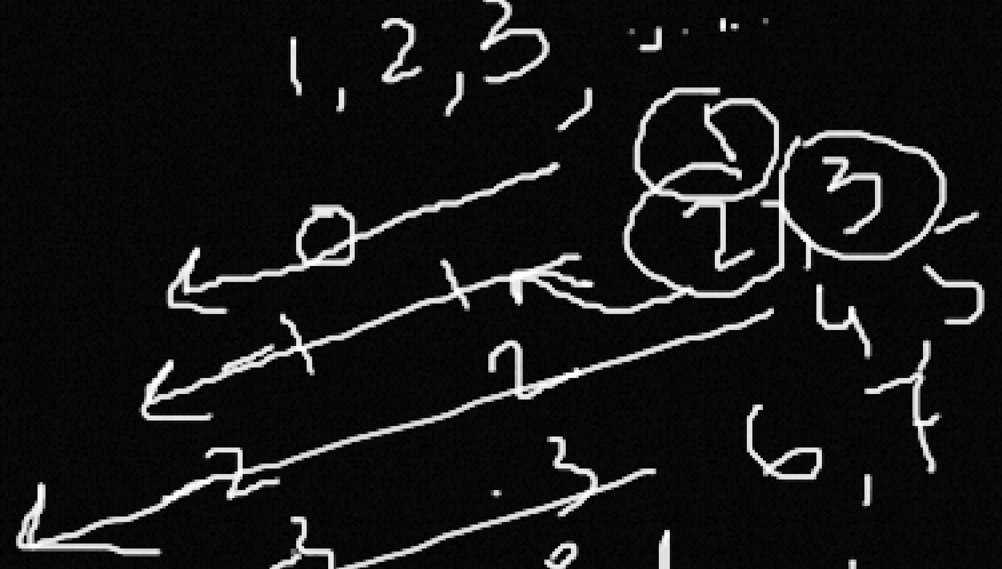
$$A \times B = \{(a, b) : a \in A, \ b \in B\}$$

eg: If $A = \{1, 2, 3\}$, $B = \{1, 2\}$

$A \times B =$



* Equivalent Set



Two sets A, B are said to be equivalent ($A \sim B$) if there exists a one-one correspondence between their elements.

eg: $\mathbb{N} \sim \mathbb{Z}$ ($1 \leftrightarrow 0$, $2 \leftrightarrow -1$, $3 \leftrightarrow 1$, $4 \leftrightarrow -2$, ...)

*** Recall previous class**

Countable Set / Enumerable set

A set X is called countable

if $X \sim \mathbb{N}$

* As discussed in previous clas

Exercise (H.W.)

$\mathbb{N} \xrightarrow{f} E$

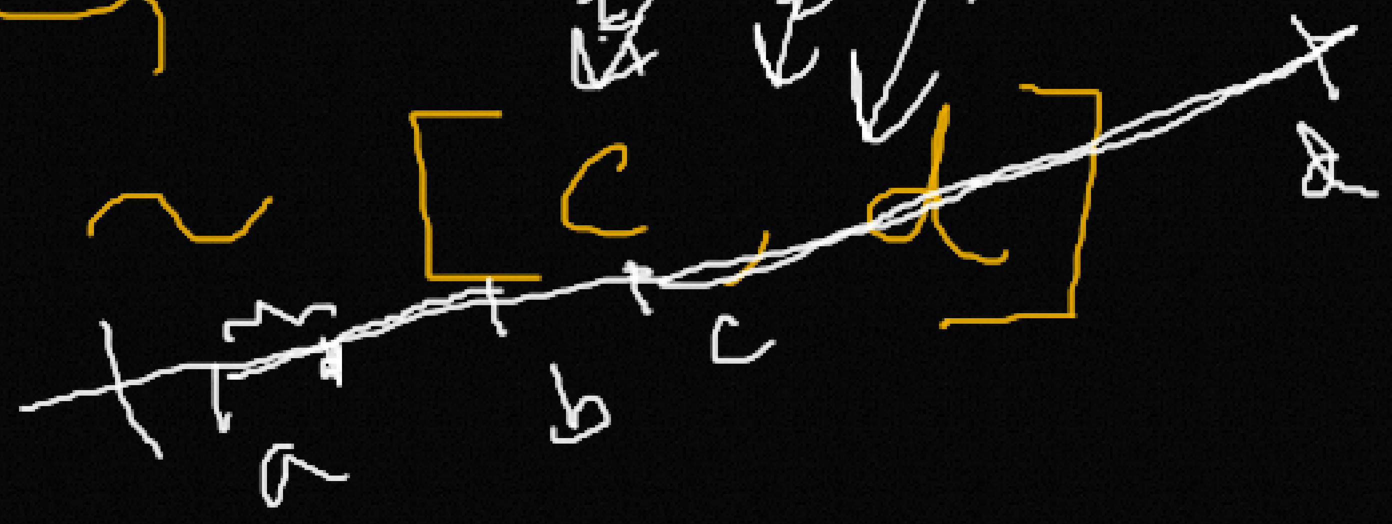
$f(t) = 2t$

1) Show that a) $\mathbb{N} \sim \{2x : x \in \mathbb{Z}\}$

Q = set of rationals

b) $\mathbb{N} \sim Q$

c) $[a, b] \sim [c, d]$



2) $A_n = [0, \frac{1}{n})$ (closed, open)

find $\bigcup_{i=1}^{b} A_i$, $\bigcup_{i=1}^{\infty} A_i$, $\bigcap_{i=1}^{\infty} A_i$

3) $A_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$

Find $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$

4) $A_n = (-n, n)$

Find $\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$

5) $A_n = \left(-4 + \frac{1}{n}, 2 - \frac{1}{n}\right)$

Find $\bigcup_{n=1}^{M} A_n$, $\bigcap_{n=1}^{M} A_n$

$\bigcup_{n=1}^{\infty} A_n$, $\bigcap_{n=1}^{\infty} A_n$

$\boxed{\text{Field}}$

A non empty set S is said to be a field if two composition (addition and multiplication) defined on S such that, for $a, b, c \in S$ we have the following:

(1) $a, b \in S \Rightarrow a+b \in S$

$\{1, 3, 5, \}$

(closed under addition)

(2) $a+b = b+a \quad \forall a, b \in S$

(addition is commutative)

(3) $(a+b)+c = a+(b+c)$

$\forall a, b, c \in S$

(addition is associative)

(4) $a + 0 = a \quad \forall a \in S$
(additive identity exists)

$\{0, 2, 4, 6, \ldots\}$

(5) for $a \in S$, $\exists\, (-a) \in S$
such that $a + (-a) = 0$
(additive inverse exists)

(6) $a, b \in S \Rightarrow ab \in S$

(7) $ab = ba \quad \forall\, a, b \in S$

(8) $a(bc) = (ab)c$

(9) $a \cdot 1 = a \quad \forall\, a \in S$

(multiplicative identity)

$\mathbb{Z}^{+} \to \{0\}, 1, 2, 3,$

(10) for $a \neq 0$, $a \in S$,

$\exists\, (a^{-1}) \in S$ s.t. $a(a^{-1}) = 1$

[multiplicative inverse exists]

11) $a, b, c \in S$

$$a(b+c) = ab + bc$$

[Multiplication is distributive under addition]

H.W.

Which of the following sets are field?

(i) $\mathbb{N}$

(ii) $\mathbb{Z}$

(iii) $\mathbb{Z}^+$

(iv) $\{2x : x \in \mathbb{Z}\}$

(v) $\mathbb{Q}$ – set of rational $0, 1, \frac{1}{2}, 2,$

(vi) $[a, b]$ $[-5, 5]$

(vii) $\mathbb{R}$